

International Journal of Modern Physics: Conference Series
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STUDIES ON A COMPLETE EXPERIMENT FOR SINGLE PSEUDOSCALAR MESON PHOTOPRODUCTION

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The complete experiment problem for photoproduction of single pseudoscalar mesons is reviewed briefly. If this problem is investigated in the context of a truncated partial wave analysis, the chance emerges to obtain a unique multipole solution measuring less polarization observables than demanded by the classical solution of the problem that is formulated in terms of spin amplitudes.

Keywords: complete experiment; polarization observables; photoproduction; truncated partial wave analysis.

PACS numbers: 11.25.Hf, 123.1K

1. Introduction

The most general expression for the amplitude describing the reaction of photoproduction of single pseudoscalar mesons reads¹

$$F_{\text{CGLN}} = i(\vec{\sigma} \cdot \hat{\epsilon}) F_1 + (\vec{\sigma} \cdot \hat{q}) \left[\vec{\sigma} \cdot (\hat{k} \times \hat{\epsilon}) \right] F_2 + i(\vec{\sigma} \cdot \hat{k}) (\hat{q} \cdot \hat{\epsilon}) F_3 \\ + i(\vec{\sigma} \cdot \hat{q}) (\hat{q} \cdot \hat{\epsilon}) F_4. \quad (1)$$

The four CGLN amplitudes $F_i(W, \theta)$, which depend on the total energy W and CMS scattering angle θ , completely describe the process. These four amplitudes are accompanied by 16 measurable polarization observables² at each point in (W, θ) . The observables are divided into the four classes: single spin (Group S), beam-target (BT), beam-recoil (BR) and target-recoil (TR).

These facts have lead to the formulation of the so called complete experiment problem³ (for a more recent publication, see Chiang and Tabakin⁴), which addresses the question of how many and which polarization observables have to be measured in order to uniquely determine the amplitudes. To start with, this problem is purely academic and disregards measurement uncertainties⁵.

2. The Complete Experiment

Additionally to CGLN amplitudes, helicity amplitudes $H_i(W, \theta)$ can be defined as well⁴, which are fully equivalent to the former. When written in terms of the H_i ,

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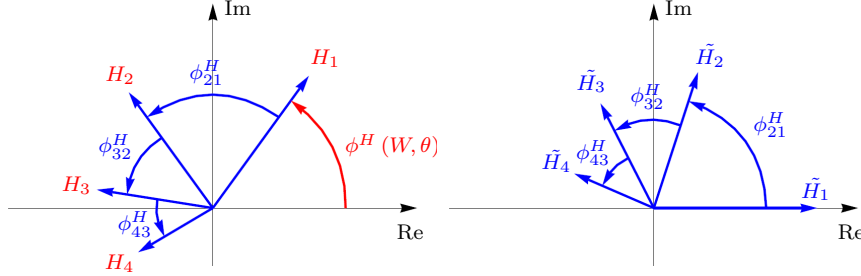


Figure 1. The left hand side shows unconstrained helicity amplitudes H_i . The right hand side shows constrained amplitudes \tilde{H}_i that can be determined solely from a complete experiment.

the profile functions $\check{\Omega}^\alpha$ defining the 16 polarization observables Ω^α via $\check{\Omega}^\alpha = \Omega^\alpha \check{\Omega}^1$ become bilinear hermitean forms⁴

$$\check{\Omega}^\alpha = \frac{1}{2} \sum_{i,j=1}^4 H_i^* \Gamma_{ij}^\alpha H_j = \frac{1}{2} \langle H | \Gamma^\alpha | H \rangle, \quad \alpha = 1, \dots, 16, \quad (2)$$

with $\{\Gamma^\alpha\}$ being a set of 16 hermitean unitary matrices. It is possible to formally invert these bilinear equations to yield⁴

$$H_i^* H_j = \frac{1}{2} \sum_{\alpha} (\Gamma_{ij}^\alpha)^* \check{\Omega}^\alpha. \quad (3)$$

The bilinear products $H_i^* H_j$ allow for the extraction of moduli $|H_i|$ as well as relative phases ϕ_{ij}^H of the helicity amplitudes (the same holds for photoproduction amplitudes defined in all other bases).

It is a central statement of Chiang and Tabakin⁴ that the amplitudes can be determined from complete sets of at least 8 observables. These sets have to contain all four Group S observables $(\frac{d\sigma}{d\Omega})_0$, Σ , T and P . The remaining four measurements must not belong to the same class of double polarization measurements. Additionally, no more than 2 observables are to be picked from the same double polarization class. The complete sets are listed⁴.

Amplitudes can only be determined from a complete experiment up to an energy and angular dependent overall phase $\phi^H(W, \theta)$, see Figure 1. Partial waves (multipoles) on the other hand can only be extracted from full production amplitudes by means of projection equations that involve angular integrations⁶. These two facts yield the result that helicity amplitudes extracted from a complete experiment up to an overall phase do not give access to partial waves (the same holds for amplitudes in all other bases). A solution to this problem consists of the performance of a truncated partial wave analysis (TPWA), which is discussed in the next section.

3. Truncated Partial Wave Analysis (TPWA)

The multipole expansion of the CGLN amplitudes is well known². Once this expansion is truncated at some finite angular momentum ℓ_{\max} and this truncation is

inserted into the polarization observables, the latter acquire an angular parametrization that is summarized by the following equations⁵

$$\tilde{\Omega}^\alpha(W, \theta) = \sin^{\beta_\alpha} \theta \sum_{k=0}^{2\ell_{\max} + \gamma_\alpha} a_k^\alpha(W) \cos^k \theta, \quad (4)$$

$$a_k^\alpha(W) = \sum_{\ell, \ell'=0}^{\ell_{\max}} \sum_{\kappa, \kappa'=1}^4 \mathcal{C}_{\ell, \ell'}^{\kappa, \kappa'} \mathcal{M}_{\ell, \kappa}^*(W) \mathcal{M}_{\ell', \kappa'}(W). \quad (5)$$

Here, the profile functions $\tilde{\Omega}^\alpha$ are expanded in finite powers of $\cos \theta$, with real expansion coefficients a_k^α and energy dependent complex multipoles $\mathcal{M}_{\ell, \kappa}(W)$. β_α and γ_α are constants depending on the observable under consideration⁵. For every finite order in $\ell_{\max} \geq 1$, $4\ell_{\max}$ complex multipoles appear in the expansion. The question of which ambiguities exist for the Group S observables in a TPWA was discussed by Omelaenko⁷ using ideas by Gersten⁸. There, the $4\ell_{\max}$ complex multipoles are exchanged for an equivalent set of complex variables α_j and β_k that are ideally suited to determine all ambiguities of the Group S observables:

$$\{\mathcal{M}_{\ell, \kappa}(W)\} \leftrightarrow \{\alpha_j, \beta_k\} = \{|\alpha_j| e^{i\phi_j}, |\beta_k| e^{i\psi_k}\}, \quad j, k = 1, \dots, 2\ell_{\max}. \quad (6)$$

Once an already existing multipole solution is used as a numerical input, the question of the arising ambiguities can be discussed in the form of a diagram⁷. Using a Bonn Gatchina⁹ solution as input, an ambiguity diagram is drawn in Figure 2 for the simple case $\ell_{\max} = 1$. The different cases of linear combinations of phases $\pm\phi_1 \pm \phi_2 \equiv \circ$ and $\pm\psi_1 \pm \psi_2 \equiv +$ are drawn in different colors. Once circles and crosses of any color almost coincide, an ambiguity of the Group S observables appears. This rule allows for the deduction of four ambiguous solutions (Fig. 2).

Once BT observables $\{E, F, G, H\}$ are plotted for those four solutions (Fig. 3), some solutions for the observables E and H cannot be distinguished.

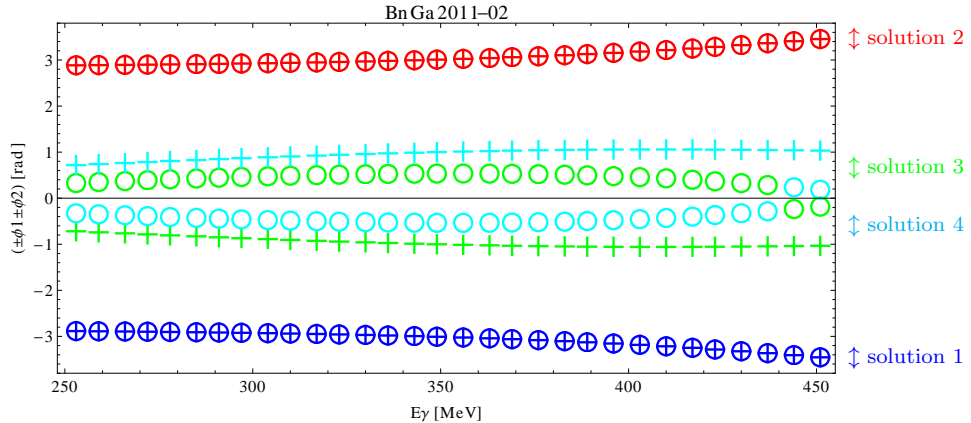


Figure 2. The ambiguity diagram for $\ell_{\max} = 1$, generated using the solution BnGa 2011-02. The solutions mentioned in the main text are indicated. A similar diagram is given by Omelaenko⁷.

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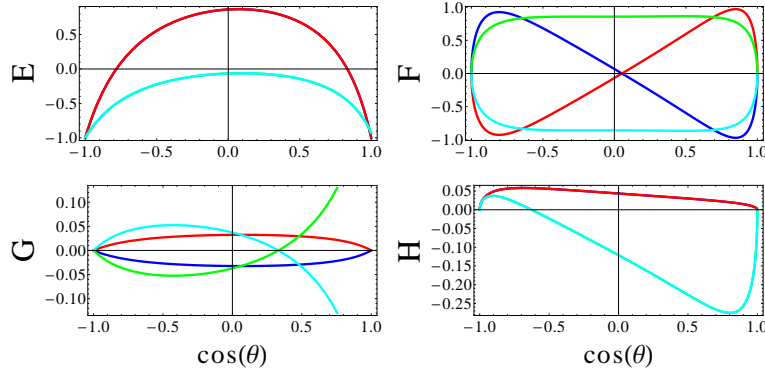


Figure 3. Preliminary plot of $\{E, F, G, H\}$ ($E_\gamma = 253$ MeV), for all four solutions.

G and F are different for all four solutions. This investigation therefore allows the postulation of sets of only 5 observables that should facilitate an unambiguous extraction of multipoles in a TPWA. This statement is consistent with Omelaenko⁷ as well as Grushin¹⁰. The reformulation of the TPWA first worked out by Omelaenko⁷ is planned to be revisited in a work that has yet to be published¹¹.

4. Summary

A set of four spin amplitudes describing pseudoscalar meson photoproduction is determined theoretically by measuring certain sets of 8 polarization observables. The undeterminable overall phase of those amplitudes denies access to partial waves. A way out of this issue consists of a truncated partial wave expansion (TPWA). First studies suggest that the multipoles of this expansion can be obtained by using even less than 8 observables.

This work was supported by the *Deutsche Forschungsgemeinschaft* within SFB/TR16.

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